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LETTER TO THE EDITOR

The dual phases of massless/massive Kalb–Ramond fields

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Abstract
We have developed the dualization of ordinary and 'Stueckelberg compensated' massive phases for Kalb–Ramond fields. The compensated phase allows the study of the interplay between spin jumping and duality. We show that spin jumping is caused by mass, while gauge symmetry is *not* necessary for this effect to occur.

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Recently, we have described a general procedure for dualizing *massless* p-forms in D dimensions, including the non-trivial limiting cases $p+1 = D$ [1,2]. The dualizing procedure has been further successfully extended to non-Abelian Kalb–Ramond (KR) field theory [3,4]. A summary of the results is that massless p-forms are dualized to $D-p-2$ forms according to the scheme

$$m = 0 : p \longleftrightarrow D - p - 2 \tag{1}$$

where the number of dynamical degrees of freedom is given by $\binom{D-2}{p}$. On the other hand, the number of dynamical degrees of freedom for the same rank but *massive* p-form is given by $\binom{D-1}{p}$. Thus, simple counting shows that massive p-forms cannot be dual to the same fields as for the massless case. In fact, they have to be dual to a $D-p-1$-form as follows:

$$m > 0 : p \longleftrightarrow D - p - 1. \tag{2}$$

As a consequence of these different schemes, the same p-form can describe fields of different spin in massless and massive phases. This phenomenon has been already observed in [5] and is known as *spin jumping*. From the dualization schemes (1) and (2) spin jumping turns out to be a general property of $p > 1$ forms. In $D = 4$ dimensions spin jumping takes place for KR ($p = 2$) and $p = 3$ fields. The latter corresponds to the limiting case $p = D - 1$ and has special properties in the massless phase as described in [1], while the former clearly exhibits spin jumping in both phases. Furthermore, KR-field theory has also been widely investigated as an alternative to the Higgs mechanism [5,6], as well as in relation to the problem of confinement in QCD and strong/weak coupling duality [7]. More recently, the generalization of the Higgs/Stueckelberg mechanism for higher-dimensional branes has been also discussed in the path integral formalism [8–10].

For these reasons in this letter we shall describe the extension of the dualization procedure for massless p-forms to the case of *massive* KR fields, with the KR field being the lowest rank p-form exhibiting spin jumping.

We shall start by reviewing the dualization of massless KR fields following [1].

One starts by introducing a parent action, which in this letter we choose to be of the first order. By this we mean an action described in terms of the independent three-form field, H, and KR gauge potential, B as follows:

$$S[H, B] = \int \mathrm{d}^4x \left[\tfrac{1}{12} H_{\lambda\mu\nu}\, H^{\lambda\mu\nu} - \tfrac{1}{6} H^{\lambda\mu\nu} \partial_{[\lambda} B_{\mu\nu]}\right]. \tag{3}$$

We shall apply the dualization procedure in a path integral framework and thus we define the generating functional as

$$Z[J] = \int [DH]\,[DB]\, \exp\left\{ - S[H, B] - \frac{g}{2} \int \mathrm{d}^4x\, B_{\mu\nu}\, J^{\mu\nu} \right\}. \tag{4}$$

This parent-generating functional is invariant under KR gauge symmetry

$$\delta H_{\lambda\mu\nu} = 0 \qquad \delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]} \tag{5}$$

provided we define an appropriate invariant integration measure and the external current $J^{\mu\nu}$ is chosen to be divergence free.

The reason we have adopted path integral formulation is that with Abelian theory it gives the same results as the algebraic approach [1] and also allows one to prove the quantum equivalence of dual theories [11]. Furthermore, it is the only formalism that allows non-Abelian extension of the dualization procedure [3]. The dualization proceeds by integrating the B field which gives the delta function:

$$\int [DB]\, \exp\left\{ - \tfrac{1}{2} \int \mathrm{d}^4x\, B_{\mu\nu}\, \left(\partial_\lambda H^{\lambda\mu\nu} - g\, J^{\mu\nu}\right) \right\} = \delta\left[\partial_\lambda H^{\lambda\mu\nu} - g\, J^{\mu\nu}\right]. \tag{6}$$

The delta function imposes classical equations of motion [12] on H which can be solved as

$$H^{\lambda\mu\nu} = \epsilon^{\lambda\mu\nu\rho} \partial_\rho \phi + g\, \partial^{[\lambda} \frac{1}{\partial^2}\, J^{\mu\nu]}. \tag{7}$$

Secondly, by integrating out the H field using the solution (7), one finds the generating functional in terms of a massless, scalar field, ϕ, which is the dual of the KR potential B

$$\begin{aligned} Z[J] &= \int [D\phi]\, \exp\left\{ -\frac{1}{12} \int \mathrm{d}^4x\, \left(\epsilon^{\lambda\mu\nu\rho} \partial_\rho \phi + g\, \partial^{[\lambda} \frac{1}{\partial^2}\, J^{\mu\nu]} \right)^2 \right\} \\ &= \int [D\phi]\, \exp\left\{ -\frac{1}{2} \int \mathrm{d}^4x\, \left[\partial_\mu \phi\, \partial^\mu \phi - \frac{g^2}{2}\, J^{\mu\nu}\, \frac{1}{\partial^2}\, J_{\mu\nu} \right] \right\}. \end{aligned} \tag{8}$$

This concludes a brief review of the known duality between a massless KR field and a scalar field using first-order formalism.

On the other hand, a *massive* KR field has a different number of dynamical degrees of freedom from a massless field and thus a massive KR field cannot be dual to a scalar field. To dualize a massive KR field we start from the parent action

$$S[H, B] = \int d^4x \left[\frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{1}{6} H^{\lambda\mu\nu} \partial_{[\lambda} B_{\mu\nu]} + \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} \right]. \tag{9}$$

The KR gauge invariance is explicitly broken and there is no *a priori* reason why the external current $J^{\mu\nu}$ should be divergence free. However, we shall still impose $\partial_\mu J^{\mu\nu} = 0$ in order to confine gauge symmetry breaking only within the mass term. We proceed by integrating out the B field

$$\begin{aligned} Z[J] &= \int [DH][DB] \exp\left\{ \int d^4x \left[\tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \tfrac{1}{2} B_{\mu\nu} \right.\right. \\ &\quad \left.\left. \times \left(\partial_\lambda H^{\lambda\mu\nu} - g J^{\mu\nu} \right) + \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu} \right] \right\} \\ &= \int [DH] \exp\left\{ \int d^4x \left[\frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} + \frac{1}{4m^2} \left(\partial_\lambda H^{\lambda\mu\nu} - g J^{\mu\nu} \right)^2 \right] \right\}. \end{aligned} \tag{10}$$

In order to obtain dual theory we express H in (10) through its Hodge Dual[4] as

$$H^{\lambda\mu\nu} \equiv m\, \epsilon^{\lambda\mu\nu\rho} A_\rho. \tag{11}$$

Inserting (11) in (10) gives the dual action of a massive vector field

$$\begin{aligned} S[A, J] &= \int d^4x \left[\frac{1}{4} \left(\epsilon^{\lambda\mu\nu\rho} \partial_\lambda A_\rho - \frac{g}{m} J^{\mu\nu} \right)^2 + \frac{m^2}{2} A_\mu A^\mu \right] \\ &\equiv \int d^4x \left[\frac{1}{4} F^{\mu\nu}(A) F_{\mu\nu}(A) - \frac{g}{6m} J^{*\mu} A_\mu + \frac{m^2}{2} A_\mu A^\mu + \frac{g^2}{4m^2} J_{\mu\nu} J^{\mu\nu} \right] \end{aligned} \tag{12}$$

where we have introduced the current $J^{*\rho} \equiv \epsilon^{\lambda\mu\nu\rho} \partial_{[\lambda} J_{\mu\nu]}$. Thus, we have shown that the massive KR field is dualized to a vector field of the Proca type as expected from scheme (2). Both fields have *three* physical degrees of freedom. Results (8) and (12) display the effect of spin jumping of the KR field from spin zero to spin one.

At this point we would like to develop a dualization procedure for a *massive, but gauge invariant*, KR theory. An effective way of achieving this goal is to apply a Stueckelberg compensation procedure [2]. In this way one can combine two properties, mass and gauge symmetry, which were mutually exclusive in (8) and (12). The reason for considering compensated theory is our intention to determine whether spin jumping is caused by the presence of mass or by gauge symmetry. For the KR-field Stueckelberg compensation amounts to the following substitution $B_{\mu\nu} \longrightarrow B_{\mu\nu} - \partial_{[\mu}\phi_{\nu]}$ where ϕ_ν is the compensating vector field transforming as $\delta\phi_\nu = \Lambda_\nu$ under KR symmetry. The Stueckelberg parent action turns out to be

$$\begin{aligned} S[H, B, \phi] = \int d^4x \left[\frac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{1}{6} H^{\lambda\mu\nu} \partial_{[\lambda} B_{\mu\nu]} + \frac{m^2}{4} \left(B_{\mu\nu} - \partial_{[\mu}\phi_{\nu]} \right)^2 \right. \\ \left. - \frac{g}{2} \left(B_{\mu\nu} - \partial_{[\mu}\phi_{\nu]} \right) J^{\mu\nu} \right]. \end{aligned} \tag{13}$$

As in the previous cases we choose a divergence-free external current, $J^{\mu\nu}$, and thus the compensator in the last term of (13) drops out. Dualization starts by integrating out the

[4] It is also possible to formulate a second-order parent Lagrangian for the massive KR field, which is the extension of the procedure in [1] and gives the same result as the above shortcut procedure.

compensator. In order to simplify integration of the Stueckelberg field, let us linearize the mass term introducing an additional field $C_{\mu\nu}$ which transforms the parent action (13) into

$$S[H, B, \phi, C] = \int \mathrm{d}^4x \left[\frac{1}{12} H_{\lambda\mu\nu}\, H^{\lambda\mu\nu} - \frac{1}{6} H^{\lambda\mu\nu} \partial_{[\lambda} B_{\mu\nu]} - \frac{1}{4} C^{\mu\nu} C_{\mu\nu} \right. \\ \left. + m\, C^{\mu\nu}\, \left(B_{\mu\nu} - \partial_{[\mu}\phi_{\nu]}\right) - \frac{g}{2} B_{\mu\nu}\, J^{\mu\nu}\right]. \tag{14}$$

Now the compensator ϕ_μ is a Lagrange multiplier and its integration gives a delta function as

$$\int [D\phi] \exp\left\{\int \mathrm{d}^4x\, m\, \phi_\nu\, \partial_\mu\, C^{\mu\nu}\right\} = \delta\left[\partial_\mu C^{\mu\nu}\right]. \tag{15}$$

The above delta function leads to the solution for the field C as

$$C^{\mu\nu} = \tfrac{1}{2}\, \epsilon^{\mu\nu\rho\sigma}\, \partial_{[\rho} A_{\sigma]}. \tag{16}$$

Subsequent integration over the B field gives the additional delta function

$$\int [DB] \exp\left\{\tfrac{1}{2} \int \mathrm{d}^4x\, B_{\mu\nu}\left(\partial_\lambda H^{\lambda\mu\nu} + m\, C^{\mu\nu} - g\, J^{\mu\nu}\right)\right\} = \delta\left[\partial_\lambda H^{\lambda\mu\nu} + m\, C^{\mu\nu} - g\, J^{\mu\nu}\right] \tag{17}$$

which, on its own, imposes an equation of motion for the field, H. Using solution (16) we find H to be

$$H^{\lambda\mu\nu} = m\, \epsilon^{\lambda\mu\nu\rho}\, \left(\partial_\rho\, \phi - A_\rho\right) + g\, \partial^{[\lambda}\, \frac{1}{\partial^2}\, J^{\mu\nu]}. \tag{18}$$

Further integration of the C field using solution (16) gives

$$\int [DC]\, \delta\left[\partial_\mu\, C^{\mu\nu}\right] \exp\left\{-\tfrac{1}{4} \int \mathrm{d}^4x\, C^{\mu\nu}\, C_{\mu\nu}\right\} \\ = \int [DA] \exp\left\{-\tfrac{1}{4} \int \mathrm{d}^4x\, F^*_{\mu\nu}(A)\, F^{*\,\mu\nu}(A)\right\} \tag{19}$$

while integration of the H field using (18) gives the final form of the dual-generating functional as

$$\int [DA]\, [DH]\, \delta\left[H^{\lambda\mu\nu} - m\, \epsilon^{\lambda\mu\nu\rho}\, \left(\partial_\rho \phi - A_\rho\right)\right] \\ \times \exp\left\{\int \mathrm{d}^4x\, [-\tfrac{1}{4}\, F^*_{\mu\nu}(A)\, F^{*\,\mu\nu}(A) + \tfrac{1}{12}\, H^{\mu\nu\rho}\, H_{\mu\nu\rho}]\right\} \\ = \int [DA][D\phi]\, \exp\left\{\int \mathrm{d}^4x \left[-\tfrac{1}{4}\, F_{\mu\nu}(A)\, F^{\mu\nu}(A) \right.\right. \\ \left.\left. + \frac{1}{2}\left(m\, \left(\partial_\mu\, \phi - A_\mu\right) + g\, \partial^{[\lambda}\, \frac{1}{\partial^2}\, J^{\mu\nu]}\right)^2\right]\right\}. \tag{20}$$

Equation (20) shows that the compensated, i.e. gauge invariant, massive, KR theory is dual to a compensated, massive, Proca theory (20). One would be tempted to believe that the presence of gauge invariance, both in the compensated and massless case, implies the same dynamics. This way of thinking suggests that (20) should be interpreted according to scheme (1) leading to the following duality:

$$B_{\mu\nu} \longleftrightarrow \phi \\ A_\mu \longleftrightarrow \phi_\mu. \tag{21}$$

Scheme (21) implies that a massive, compensated KR field is dual to a scalar field, *ergo*, it has the same dynamical content of the massless case. The same reasoning, applied to the compensated Proca theory, tells us that it should have the same dynamics as in the case of massless vector theory, i.e. two degrees of freedom. It seems that there is an inconsistency in the spin content in (20) and (13) if one follows the scheme (21), based on gauge symmetry. What happened to the spin jumping and what is the correct spin of the compensated, massive KR field? Is it determined by gauge invariance or by the presence of mass?

The answer can only be found by counting the number of physical (dynamical) degrees of freedom for the theory given by the second order form of (13). One could do this counting covariantly, *a la* Fadeev–Popov, but as already has been shown in [13] for the massless KR field, such counting turns out to be complicated. Thus, to avoid any ambiguity, we adopt a non-covariant description where the physical degrees of freedom are manifest. One could start from the equation of motion for the compensated, massive KR field

$$\partial_\mu H^{\mu\nu\rho}(B) = m^2\big(B^{\nu\rho} - \partial^{[\nu}\phi^{\rho]}\big) \tag{22}$$

$$\partial_\nu\big(B^{\nu\rho} - \partial^{[\nu}\phi^{\rho]}\big) = 0 \tag{23}$$

where (23) is a consistency condition following on from (22). Equation (23) allows us to solve the compensator ϕ_μ in terms of $B_{\lambda\mu}$ as

$$\phi_\mu = \partial^\lambda \frac{1}{\partial^2} B_{\lambda\mu}. \tag{24}$$

Inserting (24) in (23) one obtains a *non-local, gauge invariant*, field equation for B as follows:

$$\big(\partial^2 - m^2\big)\,\partial_\mu \frac{1}{\partial^2} H^{\mu\nu\rho}(B) = 0. \tag{25}$$

To extract the physical degrees of freedom contained in (25), we write separately its $(i\,0)$ and $(i\,j)$ components. The components $(i\,0)$ of (25) give the equation of motion for the transverse vector field $C^i \equiv \partial_j H^{ji0}$ as

$$\big(\partial^2 - m^2\big)\frac{1}{\partial^2}\vec{C} = 0. \tag{26}$$

Furthermore, the $(i\,j)$ components of (25) and (26) give an additional equation of motion for the 'scalar' component as

$$\big(\partial^2 - m^2\big)\,\epsilon^{ijk} H_{ijk}\big(B\big) = 0. \tag{27}$$

Equations (26) and (27) describe the dynamics of the compensated, massive KR field and show that there are *three* dynamical degrees of freedom (spin one) instead of a single (spin zero) scalar component. This is the answer to the previously raised question: a massive, compensated KR field describes a spin-one field. From the dualization result (20), one also concludes that a massive, compensated, Proca field has three degrees of freedom[5] and not two as one would naively believe. The conclusion is that in general, massive, compensated, gauge theory *is not* dynamically equivalent to a massless gauge theory, in spite of the gauge invariance of both theories. As a consequence one has to follow the dualization scheme (2) for massive p-forms. This brings us to the non-trivial conclusion that the spin jumping is caused by the *sole* presence of the mass term. Gauge symmetry is not a necessary ingredient for this effect to take place.

Let us now stress the difference between compensated and non-compensated massive theories. In the absence of a compensator, the mass term is not gauge invariant and the

[5] This conclusion can be independently confirmed by direct analysis of dynamical content in the same way as for a KR field.

consistency condition $\partial_\mu B^{\mu\nu} = 0$ relates the components of KR field so that $C^i = \partial^2 B_T^{0i}$. The equations of motion, (26) and (27), then become

$$\left(\partial^2 - m^2\right) B_T^{i0} = 0 \tag{28}$$

$$\left(\partial^2 - m^2\right) \epsilon^{ijk} H_{ijk}\left(B\right) = 0 \tag{29}$$

showing, again, three dynamical degrees of freedom as well as the gauge non-invariance of (28), which is explicitly dependent on the transverse components of the vector B^{i0}. On the other hand, in the massless case the covariant equations of motion (25) combine to give the propagation of the scalar degree of freedom

$$\partial^2 \epsilon^{ijk} H_{ijk}\left(B\right) = 0. \tag{30}$$

In this letter we have investigated massless, massive, and *massive Stueckelberg compensated* phases of KR-field theory. We have extended the dualization procedure of [1] to the latter two phases and shown their duality to ordinary and compensated Proca theories. The result of the duality procedure shows the effect of spin jumping for the KR field. While in massless and ordinary massive phase the spin content is clearly exhibited, in the compensated phase it is not so obvious and we have shown that this phase also describes a spin-one field. The advantage of the compensated phase is that one can simultaneously consider mass and gauge symmetry and study their effects on spin jumping. The result shows that the spin jumping is caused only by the presence or absence of mass.

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